Probability

P

Probability Rules

- **Addition Rule** [for simultaneous trial]
  - events are mutually exclusive: $P[A \text{ or } B] = P[A] + P[B]$, or $P[A \cup B]$
  - Events are not mutually exclusive: $P[A \text{ or } B] = P[A] + P[B] - P[A \cap B]$
- **Multiplication rule** [for successive trial]
  - Events are Independent: $P[A \& B]$ or $P[A \cap B] = P[A] \times P[B]$
  - Events are dependent: $P[A \cap B] = P\left[\frac{A}{B}\right] \cdot P[B]$ or $P\left[\frac{B}{A}\right] \cdot P[A]$

where $P[A]$ denotes probability of occurrence of events A

$P[B]$ denotes the probability of occurrence of event B

Note → Definition of Mutually, Independent and dependent events

Mutually events → Two events are said to be mutually exclusive when both the events could not happen simultaneously in single trial.

e.g. → If we toss a coin, at the same time only one event can occur whether it will be head or tail. At the same time whether we can live or dead, at the same time both the occurrence is not possible such as dead or alive.

Independent events - Two events are said to be Independent when outcome of one ~~one~~ does not affect the outcome of others.

Dependent events are those ~~when~~ in which occurrence or non-occurrence of events any one affects the probability of other events.

Mutually exclusive events

In this case $P[A \cap B] = 0$

Not mutually exclusive

$P[A \cap B] \neq 0$

In the case of Dependent event

$P[A \cap B] = P\left[\frac{A}{B}\right] \cdot P[B]$

$= P\left[\frac{B}{A}\right] \cdot P(A)$ Often

$P\left[\frac{A}{B}\right]$ means : Here B is known, it means to say the event B has already occurred.

Conditional probability in which you of one event events is known

Problem related to Independent events

problem (1)

Q → Two sets of cards with a letter on each card as following are placed into separate bags

Bag 1 → $\boxed{I}$ $\boxed{L}$ $\boxed{J}$ $\boxed{A}$ $\boxed{U}$

Bag 2 → $\boxed{L}$ $\boxed{R}$ $\boxed{H}$ $\boxed{E}$ $\boxed{e}$ $\boxed{A}$

Sara randomly picked one card from each bag. Find the probability

(a) She picked the letter J and R

[b] Both letter are L

[C] Both letters are vowels.

Solution :→ [a] At first we know that from question there are five letters in Bag 1 and six letter in Bag 2.

We have to find what is probability of getting the letter J and R when we select randomly one cards from each bag.

Fast PDF Scanner

then probability of getting J first is denoted by $P[J] = \frac{1}{5}$ (3)

probability of getting R denoted by $P[R] = \frac{1}{6}$

$P[J \text{ and } R] = P[J] \times P[R]$ [with the concept of Independent events]

or

$P.[J \cap R] = \frac{1}{5} \times \frac{1}{6}$

$= \frac{1}{30}$

[b] we have to find both the letters L

$= \frac{1}{5} \times \frac{1}{6}$

$= \frac{1}{30}$

[c] we have to find both the letter are vowels.

when we consider the Bag (1) there are three vowels in Bag (2) There are two vowels

probability of getting vowels from Bag 1 =

$P(1) = \frac{3}{5}$

probability of getting vowels from Bag 2

$P(2) = \frac{2}{6}$

probability of getting that both the letters are vowels $= \frac{3}{5} \times \frac{\cancel{2}}{\cancel{6}\,3}$

Fast PDF Scanner

Problem related to dependent events

Problem(1) The data for the promotion and academic qualification of a company is given below

Promotional status	Academic Qualification		
	MBA [A]	Non MBA [$\bar{A}$]	Total
Promoted [B]	0·14	0·26	0·40
Non promoted [$\bar{B}$]	0·21	0·39	0·60
Total	0·35	0·65	1·00

(a) Calculate the conditional probability of promotion after an MBA has been identified

[b] Calculate the conditional probability that it is an MBA when a promotional employee has been choosen.

Solution :→ At first we will find that the probability of ~~identified~~ identify the qualification MBA

$P[A] = 0·35$

the probability of identify the qualification non MBA $P[\bar{A}] = 0·65$

probability of ~~promoted~~ identifying promoted $= P(B) = 0·40$

probability of identifying Non promotion = 0.50

A probability of identifying MBA that is promoted
$P(A \cap B) = 0.14$

(i) we have to find the conditional probability of promotion after an MBA has been identified. It means to say, I have to find ~~...~~ $P\left(\frac{B}{A}\right)$

we know from formula of conditional probability

$$P[A \cap B] = P\left[\frac{B}{A}\right] \cdot P[A]$$

then $$P\left[\frac{B}{A}\right] = \frac{P[A \cap B]}{P[A]} \quad \text{— ①}$$

we know the values $P[A \cap B] = 0.14$

$P[A] = 0.35$

then we will put the values of $P[A \cap B]$ and $P(A)$ in the above equation ①

$$P\left[\frac{B}{A}\right] = \frac{0.14}{0.35}$$

$$= \frac{\cancel{14}\,2}{\cancel{35}\,5}$$

$$= 0.4$$

(b) In the second part of the question we have to find out the conditional probability an MBA when a promotional employee has been choosen. it means to say we have to find $P\left[\frac{A}{B}\right]$.

we know from formula

$$P(A \cap B) = P\left[\frac{A}{B}\right] \cdot P(B)$$

$$P\left[\frac{A}{B}\right] = \frac{P[A \cap B]}{P(B)} \quad — (2)$$

then we will put the values of $P(A \cap B)$ and $P(B)$ in equation (2)

$$P\left[\frac{A}{B}\right] = \frac{0.14}{0.40}$$

$$= \frac{\cancel{14}\,7}{\cancel{40}\,20}$$

$$= .35$$

Problem (2) The probability that a trainee will remain with a company is 0.6. The probability that an employee earns more than Rs 10,000 per month is 0.5. The probability that an employee who is a trainee remained with the company or who earns more than Rs 10,000 per month is 0.7. what is the probability that an employee earns more than Rs 10,000 per month given that he is a trainee who stayed with the company.

(2)

Solution :- The probability that a trainee who remains with the company denoted by A.

then, From question $P[A] = 0.6$

and the probability that an employee earns more than Rs 10,000 per month denoted by B

From question, $P[B] = 0.5$

the probability that an employee who is trainee remaining with the company or who earns more than Rs 10,000 per month is denoted by $P[A \cup B]$

From question $P(A \cup B) = 0.7$.

then we have to find out what is the probability that an employee earns more than Rs 10,000 per month given that he is trainee who stayed with the company. it means we have to find $P\left[\frac{B}{A}\right]$

$$P\left[\frac{B}{A}\right] = \frac{P(A \cap B)}{P[A]} \quad \text{--- (1)}$$

then at first we have to find out the values of $P(A \cap B)$

we know,

$$P(A \cup B) = P[A] + P[B] - P[A \cap B]$$

$$P(A \cap B) = P[A] + P[B] - P[A \cup B]$$

$$= 0.6 + 0.5 - 0.7$$

$$= 0.4$$

$$\text{then } P\left(\frac{B}{A}\right) = \frac{0.4}{0.6}$$

$$= \frac{4}{6}$$

$$= 0.66$$

(8)

Baye's theorem

Bayes theorem describes the probability of events based on the prior knowledge of condition that might be related to the event.

$$\cancel{P(A \cap B) = P(A) \cdot P\left[\frac{A}{B}\right]}$$

$$P[A \cap B] = P[A] \cdot P\left[\frac{B}{A}\right] \quad \text{— (1)}$$

$$P[A \cap B] = P[B] \cdot P\left[\frac{A}{B}\right] \quad \text{— (2)}$$

From equation (1) and (2)

$$P[A] \cdot P\left[\frac{B}{A}\right] = P[B] \cdot P\left[\frac{A}{B}\right]$$

$$P\left[\frac{B}{A}\right] = \frac{P[B] \cdot P\left[\frac{A}{B}\right]}{P[A]}$$

Prior probability

Posterior probability

$$P\left[\frac{B_i}{A}\right] = \frac{P(B_i) \cdot P\left[\frac{A}{B}\right]}{\cancel{P(A)} \cdot P(B_1) \cdot P\left[\frac{A}{B_1}\right] + P(B_2) \cdot P\left[\frac{A}{B_2}\right] + \dots P(B_n) \cdot P\left[\frac{A}{B_n}\right]}$$

Fast PDF Scanner

Problem :- Suppose an item is manufactured by three machines X, Y and Z. All three machines have equal capacity and are operated at the same rate. It is known that the percentages of defective items produced by X, Y and Z are 2, 7 and 12 % respectively. All the items produced by X, Y and Z are put into the bin. From this bin one item is drawn at random and is found to be defective. What is the probability that the item was produced on Y.

Solution :- Let A be the defective item.

~~then the probability of producing defective items by machine X~~

then the probability of ~~produced~~ defective items produced on machine X, machine Y and machine Z,

$$P[X] = \frac{1}{3},\ P[Y] = \frac{1}{3},\ P[Z] = \frac{1}{3}.$$

$$P\left[\frac{A}{X}\right] = 0.02,\ P\left[\frac{A}{Y}\right] = 0.07,$$

$$P\left[\frac{A}{Z}\right] = 0.12,$$

then from question we have to find

$$P\left[\frac{Y}{A}\right]$$

$$P\left[\frac{Y}{A}\right] = \frac{P\left[\frac{A}{Y}\right]\cdot P[Y]}{P[X]\cdot P\left[\frac{A}{X}\right] + P[Y]\cdot P\left[\frac{A}{Y}\right] + P[Z]\cdot P\left[\frac{A}{Z}\right]}$$

$$= \frac{\cancel{0.07}\ .07 \times \frac{1}{3}}{\frac{1}{3}\times 0.02 + \frac{1}{3}\times 0.07 + \frac{1}{3}\times 0.12}$$

$$= 0.33$$

Fast PDF Scanner